Small groups and Questionnaires (for quality control)
Introduction

• the problem in general
• the classical approach for large groups
• a transcription for small groups
The inquiry

• A questionnaire
  – questions with answers on a Likert scale

• The inquiry
  – item q&a
  – dimension: items around the same topic
  – inquiry: collection of almost independent dimensions
  – random ordering of items

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The Questionnaire

• 12 dimensions
• 3 items per dimension

Dimension:
content of lecture notes

Items
readability
understandable
badly written

The construction of such a questionnaire is a time consuming process

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## The Likert Scale

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
<th>Positive formulation</th>
<th>Negative formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>very bad</td>
<td>a</td>
<td>f</td>
</tr>
<tr>
<td>2</td>
<td>bad</td>
<td>b</td>
<td>e</td>
</tr>
<tr>
<td>3</td>
<td>close on bad</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>4</td>
<td>close on good</td>
<td>d</td>
<td>c</td>
</tr>
<tr>
<td>5</td>
<td>good</td>
<td>e</td>
<td>b</td>
</tr>
<tr>
<td>6</td>
<td>very good</td>
<td>f</td>
<td>a</td>
</tr>
</tbody>
</table>
The inquiry

• An independent agency
  – objectivity
• All at once (only one session missing data)
  – independence
• Written (Standard forms: encircling a-f per item)
  – automatic reading
• Anonymity warranted
  – no drawback

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Traditional analysis

- Scores on dimensions are summarized
  - location: mean
  - scale: standard deviation

- A decision tree is built on this summary
  - more than x dimension under 3.5
  - more than x dimensions under 2

- Reliability: cronbach alpha
- No control on outliers

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The probability model & its inverse

• Model in words
  – multivariate hypergeometric
  • sampling a box with cards (of different colors) without replacement
  – multinomial
  • a method to put the cards into the box
  – Dirichlet
  • describing the circumstances of the choice of a card

• Bayes-rule

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The probability model & its inverse for an item

Model in formulas:

- \( p(\{n_i\} | \{N_i\}, I) \)

\[
p(\{n_i\} | \{N_i\}, I) = \frac{\prod_i C_{N_i}^{n_i}}{C_n^N} \Theta(\sum_i N_i = N) \Theta(\sum_i n_i = n)
\]

- \( p(\{N_i\}\{p_i\} | I) \)

\[
p(\{N_i\}\{p_i\} | I) = N! \prod_i \frac{p_i^{N_i}}{N_i!} \Theta(\sum_i N_i = N) \Theta(\sum_i p_i = 1).
\]

- \( p(e_i a, \{p_i\} a | D a, I) \)

\[
p(e_i a, \{p_i\} a | D a, I) \propto \prod_i \frac{p_i^{n_i + \alpha_i}}{n_i!} \Theta(\sum_i n_i = n) \prod_i \frac{p_i^{e_i}}{e_i!} \Theta(\sum_i e_i = N - n)
\]
The probability model & its inverse for an dimension

Model in words:
- item 1 posterior = DMMH
- item 2 prior = posterior(item 1) = DMMH
- item 3 prior = posterior(item 2) = DMMH

DMMH belongs to the exponential family
- updating
Testing the new model

• Confirmation of the analysis done for large groups from small group model
• How reliable is the model?
• How reliable are the conclusions?
How reliable is the classical model?

• Based on the central limit theorem
  – Cronbach alpha (no direct transcription to small groups) is a measure for consistency.

• Rational argument behind this measure
  – when ranked from undesired to desired (reversing order for negatively asked questions) there is a strong correlation between items belonging to the same dimension
  – range of the ranking should be small
Range of the ranking for a dimension

- a filling in at random
- b interpreting a positively formulated question as negatively formulated
- c filling in on position

Classification of respondents
Quick & dirty

• if the range of the ordered answers in a dimension is larger than 2 then classify the dimension as non respondent

• why not 1
  – too many answers are classified as non respondent

• why not 3
  – the distinction between strongly agree and disagree a little bit should be clear
A better way to classify

• see
  – Finite Mixture and Markov Switching Models (Fruehwirth)
  – Bayesian methods for Finite Population Sampling (Ghosh & Meeden)

• adaptation to small groups is not straightforward
• going from items to dimensions is also not straightforward

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• Determine the number of respondents for a dimension
• Count n
• Determine the posterior (p & e) (updating)
• Calculate p(e)
• Communicate this for each dimension: histogram or box and whisker plot summary

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Reliability

• Simplify the statements:
  – bad---(no opinion)--- good

• Without non-respondents (no uncertainty)
  \[ \text{odds} = \frac{N_g}{N - N_g} \]

• With non-respondents (Odds becomes a RV)
  \[ N_g = n_g + e_g \quad \text{Odds} = \frac{n_g + e_g}{N - n_g - e_g} \]
Where does R come in?

• Example from the faculty of science: 5 bachelor degrees: 3 years: ± 12 courses: ± 300 questionnaires
• analysis has to be automated
• only simple commands are possible
• output can be used without modifications
aantallen<-c(16,36,15,39,12,8,8,9,18)
The sequence of the questions is standard

Analysis

The reliability control per dimension

The figures in pdf

Comments in R on the screen
Examples of reliability

No evidence 1-4
Weak evidence 4-7
Mediocre evidence 7-10
Strong evidence 10-100
Very strong evidence 100-
Discussion

• Ad hoc classification is ok for now. It was checked on large groups and it is in accordance with the construction of the questionnaire: the method should be improved for new questionnaires.

• The multi-item technique is very demanding for the author of the questions

• The Dirichlet prior is taken uniform: it contains some information (unjustified?)

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Conclusions

• The expectation value of the Odds and the reference to the evidence used in model selection, gives a good indication of the reliability of the conclusion.

• After explaining the model and it consequences, it was decided to use it temporarily only for feedback.

• The R-code did his job.