ArDec: Autoregressive-based time series decomposition in R

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Time series decomposition

Approaches:

- **non-parametric**: filtering / smoothing (eg STL, discrete wavelet transform, ...)
- **model-based**: regression, structural models, ...

Goals:

- remove “known” (non-stationary) components
- describe components of interest (seasonal, trend, ...)
Time series decomposition

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Trend & seasonality

▶ “The essential idea of trend is that it is smooth.”

▶ “A trend is a consistent pattern over time.

▶ “A trend is a long-term movement in time series data after other components have been accounted for. “

▶ “A trend is a trend, is a trend, is a trend, ...”
Trend & seasonality

- “the characteristics of a time series giving rise to spectral peaks at seasonal frequencies” [Nerlove 1964]
- “the intra-year pattern of variation which is repeated constantly or in an evolving fashion from year to year” [Shiskin et al. 1967]
- “periodic fluctuations that recur with about the same intensity each year” [Hillmer and Tiao 1982].
Problem

How to retrieve physically-relevant components?
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Method

M. West 1997

(Time series decomposition. *Biometrika* 84)

Basic concept:

\[
X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t \implies X_t = \sum_{j=1}^{p} \gamma^j_t
\]
State-space representation of AR(p) process

\[ X_t = F^T Z_t \]
\[ Z_t = GZ_{t-1} + \varepsilon_t \]

with

\[ F^T = [1 \ 0 \ ... \ 0] \]
\[ Z_t^T = [X_t \ X_{t-1} \ ... \ X_{t-p+1}] \]

\[ G = \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_p \\ 1 & 0 & \ldots & 0 \\ \vdots & 1 & \ddots & \vdots \\ 0 \end{bmatrix} \]
State-space representation of AR(p) process

\[ X_t = F^T Z_t \]
\[ Z_t = GZ_{t-1} + \varepsilon_t \]

\[ G = EAE^{-1} \quad \rightarrow \quad r_j e^{\pm iw_j} \]
\[ a = E^T F, \quad b_t = E^{-1} Z_t \]
\[ X_t = \sum_{j=1}^{p} \gamma_t^j, \quad \gamma_t^j = a^j b_t^j \]

\[ w_j = 0 \quad \rightarrow \quad \gamma_t^j = r_j \gamma_{t-j} + \nu_t \]
\[ w_j \neq 0 \quad \rightarrow \quad \gamma_t^j = 2r_j \cos(w_j) \gamma_{t-1}^j + r_j^2 \gamma_{t-2}^j + \eta_t \]
Decomposition of sea-level records with ArDec
Decomposition of sea-level records with ArDec

```r
> library(ArDec)
> coef=ardec.lm(dat)$coefficients

> coef

<table>
<thead>
<tr>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.386186446</td>
<td>0.050007536</td>
<td>0.088643459</td>
<td>0.002730004</td>
<td>0.045915250</td>
<td>-0.009539645</td>
</tr>
<tr>
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<td>0.032015897</td>
<td>-0.075378709</td>
<td>0.064847440</td>
<td>0.117705959</td>
<td>0.169322783</td>
</tr>
<tr>
<td>0.060288889</td>
<td>-0.077621640</td>
<td>-0.074880590</td>
<td>-0.012911223</td>
<td>0.010869043</td>
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<tr>
<td>-0.048499636</td>
<td>-0.002643505</td>
<td>-0.044422722</td>
<td>0.054698372</td>
<td>0.052529147</td>
<td>0.139849769</td>
</tr>
<tr>
<td>0.072803836</td>
<td>-0.081687104</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```
Decomposition of sea-level records with ArDec

> ardec(dat, coef)

<table>
<thead>
<tr>
<th>period</th>
<th>damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &quot;trend&quot;</td>
<td>&quot;0.996&quot;</td>
</tr>
<tr>
<td>2 &quot;12.089&quot;</td>
<td>&quot;0.986&quot;</td>
</tr>
<tr>
<td>3 &quot;6.000&quot;</td>
<td>&quot;0.982&quot;</td>
</tr>
</tbody>
</table>
Decomposition of sea-level records with ArDec

```r
> str(ardec.components(ardec.out))
```

List of 2
$ periodcomps:List of 2
  ..$ periods: num [1:2] 12.1 6.0
  ..$ comps : mts [1:936, 1:2] NA NA NA NA NA NA NA NA NA NA ...
  .. ..- attr(*, "dimnames")=List of 2
  .. .. ..$ : NULL
  .. .. ..$ : chr [1:2] "Series 1" "Series 2"
  .. ..- attr(*, "tsp")= num [1:3] 1928 2006 12
  .. ..- attr(*, "class")= chr [1:2] "mts" "ts"
$ trendcomp : Time-Series [1:936] from 1928 to 2006: NA NA ...
Decomposition of sea-level records with ArDec

Annual component

Semi-annual component

Trend component
Chesapeake bay
Chesapeake bay: annual components

- Baltimore
- Annapolis
- Cambridge
- Solomon’s
- Gloucester
- Hampton
- Chesapeake
- Kiptopeke
Chesapeake bay: trend components

- Baltimore
- Annapolis
- Cambridge
- Solomon's
- Gloucester
- Hampton
- Chesapeake
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Package ArDec

- implements autoregressive-based time series decomposition
- model-based, additive decomposition
- yields periods of physically-relevant (non-damped) components
- extracts flexible, time-varying estimates of such components
- option of Bayesian framework for autoregressive estimation
Thanks!