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Asymptotic Approximation for Regression Quantiles with (nearly) Root-n Error

A fundamental result for establishing asymptotic properties of regression quantiles is the well-known Bahadur Representation: $n^{1/2}(\hat{\beta}(\tau) - \beta(\tau)) = D(x)W(t) + R_n$ where $W(t)$ is a Brownian Bridge, and R_n is an error term. Unfortunately, R_n is of order $n^{-1/4}$, which might suggest that asymptotic properties are only accurate to this order. However, both simulations in (non-symmetric) regression cases and one-dimensional results justify a belief that regression quantile methods share the $O(n^{-1/2})$ accuracy of smooth statistical procedures. By expanding the finite-sample density for regression quantiles, it is shown that $n^{1/2}(\hat{\beta}(\tau) - \beta(\tau))$ is approximately normal with error $O(n^{-1/2}\,(\log(n))^a)$ at a fixed τ (under conditions). Use of this result for asymptotic inference and to obtain a uniform strong approximation in the "Hungarian" construction will be discussed.