S1-1 (invited lecture) *9.15 h*

Threshold models with fixed and random effects for ordered categorical data

Hans-Peter Piepho[1], Edith Kalka[2]

[1]Universität Hohenheim, 70593 Stuttgart, Germany
[2]Universität-Gesamthochschule Kassel, 37213 Witzenhausen, Germany

Abstract
Sensometric assessments often give rise to ordered categorical data. Depending on objective these may be analysed by a threshold model (McCullagh, 1980), in which observed frequencies in the observed ordered categories are modelled by an underlying latent continuous random variable. It is customary to assume a normal or a logistic distribution (i.e., a probit or logit link, respectively). The threshold model falls into the class of generalized linear models (GLM).

Rival methods include analysis of variance (ANOVA) and nonparameteric methods based on ranks. In the present paper we will compare ANOVA techniques to the threshold model. ANOVA has the virtue of simplicity and robustness to departures from normality and homoscedasticity. For this reason, some authors favour ANOVA over GLMs (Young et al., 1999). We hypothesise, however, that robustness properties are good only under the global null hypothesis, while mean comparisons are more strongly affected when the global null does not hold. Thus, a simulation study will be conducted to compare the size and power of group comparisons by ANOVA and by a threshold model analysis in case of group differences (violation of the global null).

In complex sampling designs (incomplete block designs with several error strata, groups of possible unbalanced experiments), one often considers mixed linear model analyses, when the normality assumption is tenable. By analogy, the threshold model may be extended by random terms in the linear predictor (Piepho, 1997). Various approximations to the likelihood have been proposed to make parameter estimation feasible (IR-REML, penalized quasi likelihood, Laplace approximation), but these have been shown to perform poorly in certain settings with binary data in small clusters (McCulloch and Searle, 2001). Recently, a versatile method for adaptive Gaussian quadrature has been suggested, which makes full maximum likelihood estimation readily accessible for mixed nonlinear models with one subject level (Pinheiro and Bates, 1995). Our simulations will therefore cover a comparison of ANOVA and quadrature-based maximum likelihood estimation of the threshold model in a mixed model set-up.

Applicability of the threshold model critically rests the appropriateness of the link function. Probit and logit link are among the most common link functions in the application threshold models. The probit link assumes normality in the latent scale. Based on recent research on mixed models with random effects having a non-normal distribution (Piepho and McCulloch, 2002), we develop a flexible class of threshold models with a nonnormal underlying latent variable to account for lack of fit of a probit link. One may transform individual random effects or larger parts of the linear predictor. Properties of maximum likelihood estimates of

the model in small samples are studied by simulation. Specifically, we will try to derive guidelines regarding the sample size necessary for the applicability of the models. Options for relaxing the equal slopes assumption are considered.

One reason for the overwhelming popularity of ANOVA in applied research is the availability of the so-called lines display of all-pairwise comparisons among a set of *t* treatments or groups. Such displays are not generally available by standard procedures when data are unbalanced and/or when a method/model other than ANOVA is used, e.g. the threshold model. We demonstrate a new method for graphically representing multiple comparisons in non-standard settings such as that faced when analysis is based on a threshold model (Piepho, 2002).

Finally, we analyse a data set from a sensometric study on the properties of different samples of curd (Kalka, 2002). We use different mixed threshold models to obtain scores for a number of traits, which are then subjected to a PCA. In the example considered, smoothing by the preceding mixed model analysis leads to an increase of the variance explained by the first to coordinates.

References

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