"Bayesian semiparametric likelihood approximations for stationary time series"

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Time series abound in many fields such as econometrics, medicine, ecology and astrophysics. Parametric models like ARMA or the more recent GARCH models dominate standard time series modeling. In particular, Bayesian time series analysis (Steel, 2008) is inherently parametric in that a completely specified likelihood function is needed. Even though nonparametric Bayesian inference has been a rapidly growing topic over the last decade, as reviewed by Hjort (2010), only very few nonparametric Bayesian approaches to time series analysis have been developed. Most notably, Carter and Kohn (1997), Gangopadhyay (1998), Choudhuri et al. (2004), Hermansen (2008), and Röver et al. (2011) used Whittle's likelihood (Whittle, 1957) for Bayesian modeling of the spectral density as the main nonparametric characteristic of stationary time series. On the other hand, frequentist time series analyses are often based on nonparametric techniques encompassing a multitude of bootstrap methods, see e.g. Härdle et al. (2003), Kirch and Politis (2011), Kreiss and Lahiri (2011).

Whittle's likelihood is an approximation of the true likelihood. Even for non-Gaussian stationary time series, which are not completely specified by their first and second-order structure, the Whittle likelihood results in asymptotically correct statistical inference in many situations. But as shown in Contreras et al. (2006), the loss of efficiency of the nonparametric approach using Whittle's likelihood can be substantial. On the other hand, parametric methods are more powerful than nonparametric methods if the observed time series is close to the considered model class but fail if the model is misspecified. Therefore, we suggest a nonparametric correction of a parametric likelihood approximation that takes advantage of the efficiency of parametric models while mitigating sensitivities through a nonparametric amendment. We use a nonparametric Bernstein polynomial prior on the spectral density with weights induced by a Dirichlet process distribution. We show that Bayesian nonparametric posterior computations can be performed via a MH-within-Gibbs sampler by making use of the Sethuraman representation of the Dirichlet process.

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