Paul Eilers

Erasmus University Medical Centre

Rotterdam, the Netherlands

p.eilers@erasmusmc.nl

Abstract

Quasi-periodic signals and the smooth complex logarithm

Many signals in nature and society are quasi-periodic, meaning that they look like sine waves with varying frequency (and amplitude). Examples are sounds of birds, bats and crickets, sunspot counts, ocean currents, and the movements of a juggler. In complex function theory the logarithm of complex variables is defined. It consists of a real component, the logarithm of the radius, and an imaginary component, the angle. Assuming a smooth time series for both components we have a very useful model for quasi-periodic data.

Fitting the smooth complex logarithm to data poses some challenges, because the model is nonlinear. Linearization with a Taylor series is straightforward. There is no serious issue for the real component, but the phase is problematic: quite good starting values are needed. The concept of the analytic signal and the use of the Hilbert transform provide an elegant solution. Smoothness is forced by difference penalties. Once a smooth phase is obtained, its derivative gives us the instantaneous frequency, which is of much interest in many applications.

The shape of the signal within each period might be more complicated than a simple sine wave. Harmonics, sine and cosine of two or three times the phase, can be added to improve the fit to the data.

I will present the model, avoiding complex technical details, and present its application to a number of real data sets.